
FUNCTIONS – REWRITING FORMULAS

The formula for the area of a square is $A = s^2$, where s is the length of a side. Since A is based on s , we can say that A is a function of s , insofar as A depends on s . We can even use the terminology: s is the input and A is the output.



The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height. Here we say that V is a function of both r and h .

To express X as a function of b means to write an equation where X is on the left side of the equation, and the right side consists of some formula containing only the variable b (and perhaps some constants).

EXAMPLE 1: **Express the side of a square as a function of its area.**

Solution: The formula $A = s^2$ expresses A as a function of s . To express the side s as a function of the area A , all we need to do is solve the formula for s :

$$s = \sqrt{A}$$

Now s is a function of A .

NOTES: First, this may seem like a trivial problem, but it's crucial when using computers. If you want to find the side of a square given the area, then you must do the algebra to solve for s — the computer cannot do it for you.

The second note is that technically, solving $A = s^2$ for s should result in $s = \pm \sqrt{A}$. But the side of a square can never be negative, so in this context it's perfectly fine to drop the “ $\pm$ ” sign and consider only the non-negative square root of A .

EXAMPLE 2: Express the area of a circle as a function of its circumference.

Solution: Normally, we think of the area of a circle as a function of its radius, namely $A = \pi r^2$, and we also consider the circumference as a function of the radius, as in $C = 2\pi r$.

To express the area, A , as a function of the circumference, C , we need to write a formula with A on the left side and some formula with C (but not r) on the right side. To accomplish this, we will remove the r from the formula $A = \pi r^2$ and insert the C from the formula $C = 2\pi r$. It's a substitution trick.

$$\begin{array}{ll} C = 2\pi r & \text{(usual circumference formula)} \\ \Rightarrow \quad r = \frac{C}{2\pi} & \text{(divide each side by } 2\pi) \end{array}$$

Now substitute this version of r into the area formula:

$$\begin{array}{ll} A = \pi r^2 & \text{(usual area formula)} \\ \Rightarrow \quad A = \pi \left(\frac{C}{2\pi} \right)^2 & \text{(since } r = \frac{C}{2\pi} \text{ from above)} \\ \Rightarrow \quad A = \pi \left(\frac{C^2}{4\pi^2} \right) & \text{(square the fraction)} \\ \Rightarrow \quad \boxed{A = \frac{C^2}{4\pi}} & \text{(cross-cancel)} \end{array}$$

EXAMPLE 3: Express the surface area of a sphere as a function of its diameter.

Solution: The standard formula for the surface area of a sphere is $S = 4\pi r^2$, a formula in which the surface area S is written as a function of the radius r . To express the area as a function of the diameter, we need to get rid of the variable r and replace it with d .

From the formula $d = 2r$, we solve for r : $r = \frac{d}{2}$. Thus,

$$S = 4\pi r^2 = \underbrace{4\pi \left(\frac{d}{2}\right)^2}_{\text{since } r = \frac{d}{2}} = 4\pi \left(\frac{d^2}{4}\right) = \pi d^2$$

Hence, $S = \pi d^2$

To help you absorb the concept of “something as a *function* of something else,” let’s summarize the formulas we’ve just seen.

$A = s^2$	A is a function of s
$V = \pi r^2 h$	V is a function of r and h
$s = \sqrt{A}$	s is a function of A
$r = \frac{C}{2\pi}$	r is a function of C
$A = \pi r^2$	A is a function of r
$A = \frac{C^2}{4\pi}$	A is a function of C
$S = 4\pi r^2$	S is a function of r
$d = 2r$	d is a function of r
$r = \frac{d}{2}$	r is a function of d
$S = \pi d^2$	S is a function of d

Homework

1. Express the area of a square as a function of its side.
2. Express the area of a square as a function of its perimeter.
3. Express the perimeter of a square as a function of its area.
4. Express the leg of a right triangle as a function of its other leg and hypotenuse.
5. Express the length of a rectangle as a function of its width and area.
6. Express the circumference of a circle as a function of its diameter.
7. Express the diameter of a circle as a function of its circumference.
8. Express the area of a circle as a function of its diameter.
9. Express the circumference of a circle as a function of its area.
10. Express the volume of a cylinder as a function of its height and radius.
11. Express the radius of a cylinder as a function of its volume and height.
12. Express the radius of a sphere as a function of its surface area.
13. Express the volume of a sphere as a function of its diameter.
($V = \frac{4}{3}\pi r^3$)
14. Express the radius of a sphere as a function of its volume.
15. Express the volume of a sphere as a function of its surface area.
16. Express the surface area of a sphere as a function of its volume.



Review Problems

17. a. Express the volume of a cube as a function of its surface area.
 b. Express the surface area of a cube as a function of its volume.
18. In the formula for the mean, $\bar{x} = \frac{x_1 + x_2}{2}$, express x_1 as a function of x_2 and $\bar{x}$.
19. True/False:
- a. $A = \pi r^2$ gives the radius of a circle as a function of its area.
 b. $S = \pi d^2$ gives the surface area of a sphere as a function of its diameter.
 c. If $G = \sqrt{2}h^5$, it's possible to write h as a function of G .

Solutions

1. $A = s^2$
2. $A = s^2$; but $P = 4s \Rightarrow s = \frac{P}{4}$; substituting into the area formula:

$$A = \left(\frac{P}{4}\right)^2 \Rightarrow A = \frac{P^2}{16}.$$
3. $P = 4s$; but $A = s^2 \Rightarrow s = \sqrt{A}$; substituting gives $P = 4\sqrt{A}$.
4. Since, $(\text{leg}1)^2 + (\text{leg}2)^2 = h^2$, it follows that $\text{leg}1 = \sqrt{h^2 - (\text{leg}2)^2}$.
5. $l = \frac{A}{w}$

6. $C = 2\pi r$; $d = 2r \Rightarrow r = \frac{d}{2}$. Therefore, $C = 2\pi\left(\frac{d}{2}\right) \Rightarrow C = \pi d$.

7. $d = \frac{C}{\pi}$

8. $A = \pi r^2$; $r = \frac{d}{2}$. Therefore, $A = \pi\left(\frac{d}{2}\right)^2 \Rightarrow A = \frac{\pi d^2}{4}$.

9. $C = 2\pi r$; $A = \pi r^2 \Rightarrow r = \sqrt{\frac{A}{\pi}}$. Thus, $C = 2\pi\sqrt{\frac{A}{\pi}}$.

10. $V = \pi r^2 h$ 11. $r = \sqrt{\frac{V}{\pi h}}$ 12. $r = \sqrt{\frac{S}{4\pi}}$

13. $V = \frac{4}{3}\pi r^3$; $r = \frac{d}{2}$. Therefore, $V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \Rightarrow V = \frac{\pi d^3}{6}$.

14. $r = \sqrt[3]{\frac{3V}{4\pi}}$ 15. $V = \frac{4}{3}\pi\left(\frac{S}{4\pi}\right)^{3/2}$ 16. $S = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$

17. a. $V = \left(\frac{A}{6}\right)^{3/2}$ b. $A = 6V^{2/3}$

18. Solving the mean formula for x_1 gives $x_1 = 2\bar{x} - x_2$.

19. a. F b. T c. T

**“On matters of style,
swim with the current;
on matters of principle,
stand like a rock.”**

Thomas Jefferson